Exam. Code : 103201
Subject Code : 1025

B.A./B.Sc. 1st Semester

MATHEMATICS

Paper—I (Algebra)

Time Allowed—Three Hours] [Maximum Marks—50

Note :— Attempt **FIVE** questions in all, selecting at least **ONE** question from each section. The **fifth** question may be attempted from any section. All questions carry equal marks.

SECTION—A

1. (a) (i) Prove that every skew-symmetric matrix of odd order has rank less than its order.

(ii) If A is a skew-symmetric matrix, then show that $\rho(A) \geq 2$.

(b) Find the rank of the matrix

$$\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

by reducing it to echelon form.

2. (a) Discuss for all values of k, the system of equations :

$$(3k - 8)x + 3y + 3z = 0$$
$$3x + (3k - 8)y + 3z = 0$$
$$3x + 3y + (3k - 8)z = 0$$

as regards the nature of solutions.

(b) Test for consistency :

$$3x + 3y + z = 9$$
$$x + 2y + 3z = 6$$
$$3x + y + 2z = 8$$

If consistent, solve for x, y, z by finding the inverse of the coefficient matrix.

SECTION—B

3. (a) Prove that λ is an eigen value of n-rowed square matrix A over a field F if and only if $|A - \lambda I| = 0$.

(b) Find the characteristic roots and spectrum of the matrix $\begin{bmatrix} 2 & 3 & 11 \\ 0 & 3 & 17 \\ 0 & 0 & -2 \end{bmatrix}$.

4. (a) Verify Cayley-Hamilton theorem for the matrix A and find A^{-1} where :

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 1 & 5 & 12 \end{bmatrix}.$$

(b) Write down the quadratic form corresponding to the symmetric matrix $\begin{bmatrix} 2 & 4 & 5 \\ 4 & 3 & 1 \\ 5 & 1 & 1 \end{bmatrix}$.

SECTION—C

5. (a) Prove that the range of values of two congruent quadratic forms are the same.

(b) Reduce the following quadratic forms to sum of squares by linear transformation :

$$2x^2 + 9y^2 + 6z^2 + 8xy + 8yz + 6zx.$$

6. (a) Reduce the following to canonical form and find the rank and index :

$$3x_1^2 - 3x_2^2 - 5x_3^2 - 2x_1x_2 - 6x_2x_3 - 6x_3x_4.$$

(b) Show that the following form is indefinite and find the set of values of the variables for which they assume positive, negative and zero values :

$$11x^2 + 14xy + 8yz + 14xz.$$

SECTION—D

7. (a) Find the condition that the sum of two roots of $f(x) = a_0x^3 + a_1x^2 + a_2x + a_3 = 0$ should also be a root. Verify the same condition for the equation $8x^3 - 8x^2 + 1 = 0$ and solve it.

(b) Form an equation whose roots are m times those of the given equation. Also transform the equation $2x^3 - 15x^2 + 24x - 7 = 0$ in which the third term is missing.

8. (a) Discuss the nature of roots of the cubic $x^3 - 6x + 4 = 0$ and solve it.

(b) Solve by Descart's method the following :

$$x^4 - 2x^3 + 4x^2 + 6x - 21 = 0.$$